3.8 Derivatives of inverse trig functions

Derivative of the Arcsine

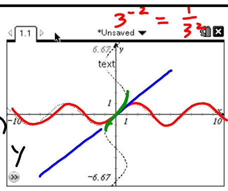
$3^{-2} = \frac{1}{3^2}$

not an exponent

$y = \sin^{-1}(x)$ *means* $x = \sin(y)$

solve for y

$y = \sin x$ swap x & y



restrict the range to make y=arcsine(x) a function

$\sin^{-1}(\frac{1}{2}) = \frac{\pi}{6}$

$-\frac{\pi}{2} \le \sin^{-1}x \le \frac{\pi}{2}$ range

$\sin^{-1} 2$ dne

$-1 \le x \le 1$ domain

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$y = \sin^{-1}x$ means $x = \sin y$

$1 = y' \cos y$

$y' = \frac{1}{\cos y}$

$\sin^2 y + \cos^2 y = 1$

$\cos y = \sqrt{1 - \sin^2 y}$

$\cos y = \sqrt{1 - x^2}$

$y' = \frac{1}{\sqrt{1-x^2}}$

$$\boxed{\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}}$$

$$\frac{d}{dx}\sin^{-1}(3x) = 3 \cdot \frac{1}{\sqrt{1-(3x)^2}}$$

$$\boxed{\frac{d}{dx}\sin^{-1}u = \frac{du}{dx} \cdot \frac{1}{\sqrt{1-u^2}}}$$

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$$\frac{d}{dx}\left(\sin^{-1}(x^2)\right) = 2x \cdot \frac{1}{\sqrt{1-(x^2)^2}} = \frac{2x}{\sqrt{1-x^4}}$$

$$\frac{d}{dx}\left(\sin^{-1}\frac{\sqrt{x}}{3}\right) = \frac{\frac{1}{2}x^{-\frac{1}{2}}}{3} \cdot \frac{1}{\sqrt{1-\left(\frac{\sqrt{x}}{3}\right)^2}} = \frac{1}{6\sqrt{x}} \cdot \frac{1}{\sqrt{1-\frac{x}{9}}}$$

$\frac{x^{\frac{1}{2}}}{3}$

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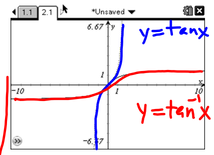
Derivative of the Arctangent

$$\boxed{\frac{d}{dx}\tan^{-1}u = \frac{du}{dx} \cdot \frac{1}{1+u^2}}$$



What is the range of y=Arctan(x) ?

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A particle moves along the x-axis so that its positon at any time t>=0 is $x(t)=\tan^{-1}\sqrt{t}$ What is the velocity of the particle when t=16?

$$v(t)=x'(t)=\frac{1}{2}t^{-1/2}\cdot\frac{1}{1+(\sqrt{t})^2}$$

$$x'(t)=\frac{1}{2\sqrt{t}}\cdot\frac{1}{1+t}$$

$$x'(16)=\frac{1}{2\sqrt{16}}\cdot\frac{1}{1+16}=\frac{1}{8}\cdot\frac{1}{17}=\frac{1}{136}$$

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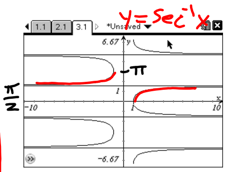
Derivative of the Arcsecant

$$\boxed{\frac{d}{dx}\sec^{-1}u=\frac{du}{dx}\,\frac{1}{|u|\sqrt{u^2-1}}}$$



restrict the range to make y=Arcsec(x) a function

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$$\frac{d}{dx}\sec^{-1}(5x^4)=20x^3\cdot\frac{1}{|5x^4|\sqrt{(5x^4)^2-1}}$$

$$=\frac{4}{x\sqrt{25x^8-1}}$$

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Derivatives of the other three

$\cos^{-1}x=\frac{\pi}{2}-\sin^{-1}x$

$\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$

$$\frac{d}{dx}\cos^{-1}x=-\frac{d}{dx}\sin^{-1}x=\frac{-1}{\sqrt{1-x^2}}$$

$\cot^{-1}x=\frac{\pi}{2}-\tan^{-1}x$

$$\frac{d}{dx}\cot^{-1}x=\frac{-1}{1+x^2}$$

$\csc^{-1}x=\frac{\pi}{2}-\sec^{-1}x$

$$\frac{d}{dx}\csc^{-1}x=\frac{-1}{|x|\sqrt{x^2-1}}$$

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Derivative of an inverse function

If *f* and *g* are inverse functions then $g'(x) = \frac{1}{f'(g(x))}$

$g(x) = f^{-1}(x)$

$g'(x) = \frac{1}{f'(y)}$

$y = g(x)$

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